edexcel

Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Mechanics 4 (6680/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2016
Publications Code 6680_01_1606_MS
All the material in this publication is copyright
© Pearson Education Ltd 2016

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

  Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

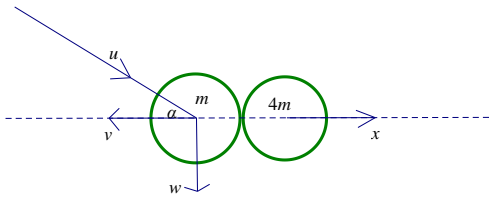
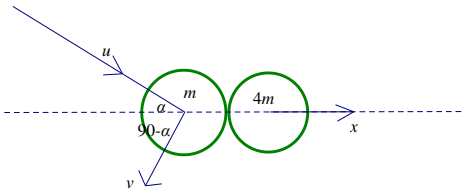
  NEL Newton's Experimental Law (Newton's Law of Impact)

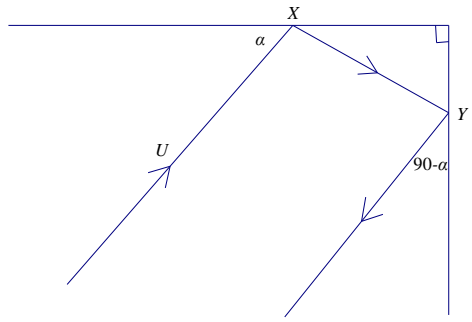
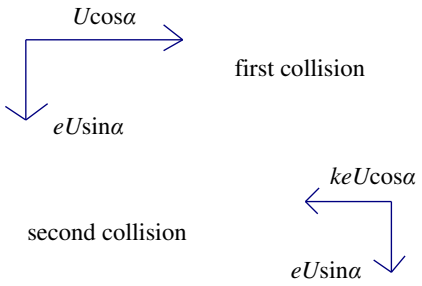
  HL Hooke's Law

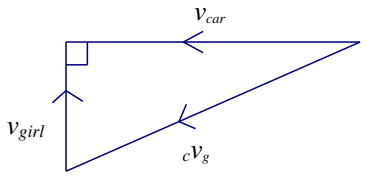
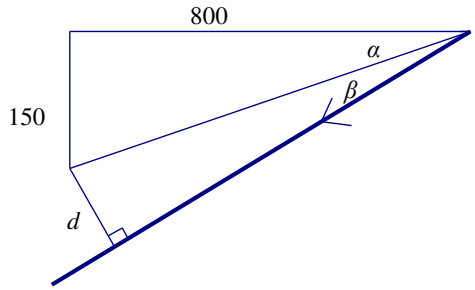
  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

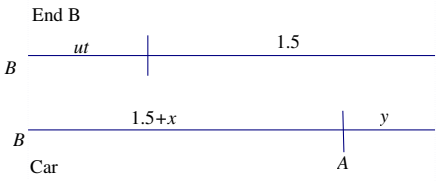
Q	Scheme	Marks	Notes
1.			
	Along line of centres:		
	Con of mom: $mu\cos\alpha = 4mx - mv$	M1	$mu\cos\alpha = 4mx - mv\cos\beta$ or $mu\cos\alpha = 4mx - mv\sin\alpha$ Need to see all 3 terms, but condone sign errors & trig. confusion
	$(u\cos\alpha = 4x - v)$	A1	$(u\cos\alpha = 4x - v\cos\beta)$ $(u\cos\alpha = 4x - v\sin\alpha)$
	NLR: $\frac{1}{2}u\cos\alpha = x + v$	M1	$\frac{1}{2}u\cos\alpha = x + v\cos\beta$ $\frac{1}{2}u\cos\alpha = x + v\sin\alpha$ Must be used the right way round, but condone sign errors & consistent trig. confusion
	$(2u\cos\alpha = 4x + 4v)$	A1	$(2u\cos\alpha = 4x + 4v\cos\beta)$ $(2u\cos\alpha = 4x + 4v\sin\alpha)$
	$(5v = u\cos\alpha)$		$(5v\tan\alpha = u)$ $(u\cos\alpha = 5v\cos\beta)$
	Perp to line of centres: no change to velocity so vel $= w = u\sin\alpha$	B 1 (A1)	$v\cos\alpha = u\sin\alpha$ $(v = u\tan\alpha)$
	Deflected through 90° $\left(\tan\alpha = \frac{v}{w}\right)$	B1	90° used correctly. E.g. use of $90 - \alpha$ in an equation $(\tan\alpha \times \tan\beta = 1)$
	$\tan\alpha = \dfrac{\frac{1}{5}u\cos\alpha}{u\sin\alpha}$	M1	$5u\tan^2\alpha = u$ Form equation in α
	$\tan^2\alpha = \frac{1}{5}$ $\tan\alpha = \sqrt{\frac{1}{5}}$ or 0.4472....	A1	(0.45 or better)
		[8]	

Q	Scheme	Marks	Notes
2			
	First impact:		
	Component parallel to wall: $=U\cos\alpha$	B1	
	Perp to wall: NLR: $eU\sin\alpha$	M1	Correct use of impact law Condone trig. confusion
		A1	
	Second impact:		
	parallel to wall vel after $=eU\sin\alpha$	B1	In terms of U and α
	Perp to wall $ke\times U\cos\alpha$	B1	In terms of U and α
	Direction at $(90-\alpha)$ to the wall	B1	Seen or implied
	$\Rightarrow \tan(90-\alpha)=\dfrac{keU\cos\alpha}{Ue\sin\alpha}$ or $\tan\alpha=\dfrac{eU\sin\alpha}{keU\cos\alpha}$	M1	
	$\cot\alpha=k\cot\alpha$ or $\tan\alpha=\dfrac{1}{k}\tan\alpha$	A1	Equation in k and α
	$k=1$	A1	From correct work only
		[9]	
	NB:A candidate who makes a false assumption about an angle α in triangle CXY can score a maximum B1B1B1 B0B0 B1 M1 A0 (6/8)		

Q	Scheme	Marks	Notes
3			
(a)	${}_c\mathbf{v}_g = \mathbf{v}_c - \mathbf{v}_g$  	B1	Correct vector triangle seen or implied e.g. sight of ${}_c\mathbf{v}_g = \begin{pmatrix}-20\\-5\end{pmatrix}$ or correct final bearing
	$\tan\theta = \frac{20}{5} \quad \theta = 75.96°$ or $\tan\theta = \frac{5}{20}, \quad \theta = 14.04°$	M1	Use trig. to find a relevant angle
		A1	Angle correct
	Direction is $256°$	A1	
	Mag $= \sqrt{20^2 + 5^5}$	M1	
	$= \sqrt{425} \quad (= 20.61...)$ (m s^{-1})	A1	$5\sqrt{17}$ Accept 21
		(6)	
(b)	 		
	Dist apart at noon $= \sqrt{150^2 + 800^2} \left(= \sqrt{662500} = 813.94...\right)$	M1	
	$\tan\alpha = \frac{150}{800}$	M1	Use trig to find α
	$\alpha = \tan^{-1}\left(\frac{150}{800}\right), \quad (\alpha = 10.619)$	A1	Correct equation in α
	$\beta = 14.04 - 10.619 = 3.420...$	M1 (A1)	Correct strategy for β Their θ – their α
	$\sin\beta = \frac{d}{\sqrt{662500}}$	M1	Use trig. to find d
		A1	Correct unsimplified expression
	$d = \sqrt{662500}\sin 3.42 \quad = 48.5...$ m	A1	or exact answer $\frac{200}{\sqrt{17}}$ Accept 49 or better
		(7)	
		[13]	
			See over for vector alternative

Q	Scheme	Marks	Notes
3balt	Relative position $\begin{pmatrix}800-20t\\150-5t\end{pmatrix}$	M1	By subtraction
	Distance $d^2=(800-20t)^2+(150-5t)^2$	M1	Correct use of Pythagoras' theorem
	$\left(=425t^2-33500t+662500\right)$	A1	Correct unsimplified expression for d or d^2
	$-40(800-20t)-10(150-5t)$	M1	Differentiate
	$(850t-33500=0)$	M1 (A1)	Equate to zero and solve for *t*.
	$t=\frac{670}{17}$, 39.4(s)	A1	
	$\Rightarrow d=48.5$ (m)	A1	
3balt	Relative position $\begin{pmatrix}800-20t\\150-5t\end{pmatrix}$	M1	By subtraction
	Distance $d^2=(800-20t)^2+(150-5t)^2$	M1	Correct use of Pythagoras' theorem
	$\left(=425t^2-33500t+662500\right)$	A1	Correct unsimplified expression for d or d^2
	$\begin{pmatrix}800-20t\\150-5t\end{pmatrix}\bullet\begin{pmatrix}-20\\-5\end{pmatrix}=0$	M1	Use scalar product with ${}_c\mathbf{v}_g=\begin{pmatrix}-20\\-5\end{pmatrix}$
	$-20(800-20t)-5(150-5t)=0$	M1	Equate scalar product to zero and solve for *t*
	$t=\frac{670}{17}$, 39.4(s)	A1	
	$\Rightarrow d=48.5$ (m)	A1	
4			

Q	Scheme	Marks	Notes
(a)	NL2: $9\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -24v - 16x$	M1	Requires all 3 terms but condone sign errors. Condone $\dot{x}$ for v. Must be dimensionally correct.
		A1	Correct unsimplified equation with v. Accept with $9a$ in which case accept $\pm$
	$9\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 24\dfrac{\mathrm{d}x}{\mathrm{d}t} + 16x = 0$	M1	Substitute for v (seen anywhere) to form equation in x and t only
		A1	**Given answer** as printed - from correct solution.
	NB: If never see v used, max score 1/4	(4)	
(b)	$\ddot{x} = 0 \Rightarrow 16x = -24\dot{x}$	M1	$\ddot{x} = 0$ used Accept equivalent forms
	$16d = 24 \times 8\mathrm{e}^{-1}$	A1	$\dot{x}$ substituted correctly
	$d = 12\mathrm{e}^{-1}$	A1	4.4 or better
		(3)	
(b) alt		M1	Differentiate twice and find A and B. Condone use of $t = \dfrac{3}{4}$, $\dot{x} = 8\mathrm{e}^{-1}$
	$x = e^{-\frac{4}{3}t}(8t + 6)$	A1	
	$d = 12\mathrm{e}^{-1}$	A1	
		(3)	
(c)	$\dot{x} = -\dfrac{4}{3}\mathrm{e}^{-\frac{4}{3}t}(At + B) + A\mathrm{e}^{-\frac{4}{3}t}$	M1	Differentiate the given general solution using the product rule
		A1	Correct unsimplified
	$-8\mathrm{e}^{-1} = -\dfrac{4}{3}\mathrm{e}^{-1}\left(\dfrac{3}{4}A + B\right) + A\mathrm{e}^{-1}$	M1	Use $t = \dfrac{3}{4}$, $\dot{x} = -8\mathrm{e}^{-1}$
	$-8 = -A - \dfrac{4}{3}B + A$		
	$B = 6$	A1	
	The first 4 marks for (c) are available when seen		
	$\left(x = \mathrm{e}^{-\frac{4}{3}t}(At + 6)\right)$		
	$t = 0 \;\; x = 6$	B1ft	their B
		(5)	
		[12]	

Q	Scheme	Marks	Notes
5 (a)(i)	Dist moved by end $B = ut$ Dist of end B from initial position of car $= 1.5 + ut$ Length of rope $= 1.5 + x$ $\therefore 1.5 + ut = 1.5 + x + y$	M1	
	$\Rightarrow x + y = ut$	A1	**Given answer**
		(2)	
(ii)	$T = \dfrac{27x}{1.5} = 18x$	B1	
	Eqn of motion for car: $0.5\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} = 18x$	M1	Must start out with $\ddot{y}$
	$x + y = ut \qquad -\ddot{x} = \ddot{y}$	A1	Correct substitution for $\ddot{y}$ - must be explained
	$\left(\ddot{x} = -36x\right) \qquad \Rightarrow \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 36x = 0$	A1	No errors seen. **Given answer** as printed
		(4)	
(b)	$x = a\sin 6t$	M1	
	$x = 0 \quad \sin 6t = 0$	M1	Find the value of t when $x = 0$ or substitute $t = \dfrac{\pi}{6}$
	$6t = \pi \qquad t = \dfrac{\pi}{6}$	A1	No errors seen **Given answer**
		(3)	
(c)	$\dot{x} = 6a\cos 6t$	M1	Differentiate their x
	$\Rightarrow \dot{y} = u - 6a\cos 6t \ , \qquad u = 6a$	M1 (A1)	use $\dot{y} = u - \dot{x}$ and $\begin{cases} t = 0,\ \dot{y} = 0 \\ \text{or } t = \dfrac{\pi}{12} \end{cases}$
	$t = \dfrac{\pi}{12} \quad \dot{y} = u - u\cos\dfrac{6\pi}{12} = u$	A1	
		(3)	
(d)	String slack when $t = \dfrac{\pi}{6} \qquad \dot{y} = u - u\cos\pi$	M1	Find speed of car when string goes slack.
	$= 2u$	A1	
	Time $= 1.5 \div u = \dfrac{3}{2u}$	B1	Time to close gap = $\dfrac{1.5}{2u - u}$
	Total distance travelled $= \left(\dfrac{\pi}{6} + \dfrac{3}{2u}\right)u + 1.5$	M1	Distance travelled by B + 1.5
	$= \dfrac{\pi u}{6} + 3$	A1	
		(5)	
		[17]	

Q	Scheme	Marks	Notes
6 (a)	GPE $= 4mgl\cos 2\theta + 2mg \times 2l\cos 2\theta$	M1	GPE of rod + particle (relative to a fixed point)
	$= 8mgl\cos 2\theta$	A1	Correct total
	Length of string $= 2\times 2l\sin\theta$ EPE $= \dfrac{kmg}{4l}\left(4l\sin\theta - 2l\right)^2$	M1	Use of EPE $= \dfrac{\lambda x^2}{2a}$
	$= kmgl\left(2\sin\theta - 1\right)^2$	A1	
	$kmgl\left(4\sin^2\theta - 4\sin\theta + 1\right)$ $+ 8mgl\left(1 - 2\sin^2\theta\right) + \text{const}$	M1	Total PE expressed in $\sin\theta$
	$= 4mgl\left\{\left(k-4\right)\sin^2\theta - k\sin\theta\right\} + \text{const}$	A1	**Given answer** as printed
		(6)	
(b)	$V' = \left(4mgl\right)\left\{\left(k-4\right)2\sin\theta\cos\theta - k\cos\theta\right\}$	M1	Differentiate V – condone errors but do not accept integration
	$\Rightarrow V' = 0 \quad \left(k-4\right)2\sin\theta\cos\theta - k\cos\theta = 0$	DM1	Set $V' = 0$ and solve for $\text{trig}\left(\theta\right) = \text{f}\left(k\right)$ Dependent on the first M1
	$4\cos\theta\left\{\left(2k-8\right)\sin\theta - k\right\} = 0$		
	$\sin\theta = \dfrac{k}{2k-8}$	A1	$\cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$ need not be seen
	$\dfrac{\pi}{6} < \theta < \dfrac{\pi}{2} \Rightarrow \left(\dfrac{1}{2} <\right) \dfrac{k}{2k-8} < 1$	B1ft	ft on their $\sin\theta$
	$\Rightarrow k < 2k - 8$	M1	Solve right hand inequality for k
	$\therefore k > 8$	A1	**Given answer**
		(6)	
(c)	$V'' = \left(4mgl\right)\left(12\cos 2\theta + 10\sin\theta\right)$	M1	Substitute $k = 10$ and find second derivative of V
	$V'' = 8\left(mgl\right)\left(6\cos 2\theta + 5\sin\theta\right)$	A1	Any equivalent form
	$\sin\theta = \dfrac{10}{12} = \dfrac{5}{6} \qquad \cos 2\theta = 1 - 2\times\dfrac{25}{36} = -\dfrac{7}{18}$	DM1	Dependent on the previous M1 Substitute their trig. values Need to be considering the whole of V''
	$V'' = 8mgl\left(-\dfrac{42}{18} + \dfrac{25}{6}\right) = mgl\dfrac{44}{3} > 0$		Accept 14.3 or better
	$\therefore$ (V min and) equilibrium is stable.	A1	With no errors seen
		(4)	
		[16]	

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom